Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 23 Issue 6 Version 1.0 Year 2023
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Quasi-P-Normal and N-Power Class Q Composite Multiplication Operatos on the Complex Hilbert Space

By M. Nithya, Dr. K. Bhuvaneswari & Dr. S. Senthil
Mother Teresa Women's University

Abstract- In this paper, the condition under which composite multiplication operators on $L^2(\mu)$-space become Quasi-P-Normal operators and n-Power class Q operator have been obtained in terms of radon-nikodym derivative f_0.

Keywords: composite multiplication operator, conditional expectation, Quasi-p-normal, multiplication operator, class Q operator.

GJSFR-F Classification: MSC 2010: 47B33, 47B20, 46C05.

Strictly as per the compliance and regulations of:

eJournal Print Journal epaper open standard 3d virtual journal

Notes

Quasi-P-Normal and N-Power Class Q Composite Multiplication Operatos on the Complex Hilbert Space

M. Nithya α, Dr. K. Bhuvaneswari σ & Dr. S. Senthil ρ

Abstract- In this paper, the condition under which composite multiplication operators on $L^2(\mu)$ -space become Quasi-P-Normal operators and n-Power class Q operator have been obtained in terms of radon-nikodym derivative f_0.

Keywords: composite multiplication operator, conditional expectation, Quasi-p-normal, multiplication operator, class Q operator.

I. Introduction

Let (X,Σ,μ) be a σ-finite measure space. Then a mapping T from X into X is said to be a measurable transformation if $T^{-1}(E)\in\Sigma$ for every $E\in\sum$. A measurable transformation T is said to be non-singular if $\mu(T^{-1}(E))=0$ whenever $\mu(E)=0$. If T is non-singular then the measure μT^{-1} defined as $\mu T^{-1}(E)=\mu(T^{-1}(E))$ for every E in Σ, is an absolutely continuous measure on Σ with respect to μ.Since μ is a σ-finite measure, then by the Radon-Nikodym theorem, there exists a non-negative function f_0 in $L^1(\mu)$ such that $\mu T^{-1}(E)=\int_E f_0\, d\mu$ for every $E\in\Sigma$. The function f_0 is called the Radon-Nikodym derivative of μT^{-1} with respect to μ.

Every non- singular measurable transformation T from X into itself induces a linear transformation C_T on $L^p(\mu)$ defined as $C_T f = f\circ T$ for every f in $L^p(\mu)$. In case C_T is continuous from $L^p(\mu)$ into itself, then it is called a composition operator on $L^p(\mu)$ induced by T. We restrict our study of the composition operators on $L^2(\mu)$ which has Hilbert space structure. If u is an essentially bounded complex-valued measurable function on X, then the mapping M_u on $L^2(\mu)$ defined by $M_u f = u\cdot f$, is a continuous operator with range in $L^2(\mu)$. The operator M_u is known as the multiplication operator induced by u.

A composite multiplication operator is linear transformation acting on a set of complex valued Σ measurable functions f of the form

Author α σ: Department of Mathematics, Mother Teresa Women's University, Kodaikanal, Tamilnadu, India.
Author ρ: Department of Economics and Statistics, ICDS, Collectorate, Dindigul, Tamilnadu, India. e-mail: senthilsnc83@gmail.com

$$M_{u,T}(f) = C_T M_u(f) = u \circ T \ f \circ T$$

Where u is a complex valued, Σ measurable function. In case $u = 1$ almost everywhere, $M_{u,T}$ becomes a composition operator, denoted by C_T.

In the study considered is the using conditional expectation of composite multiplication operator on L^2-spaces. For each $f \in L^p(X, \Sigma, \mu)$, $1 \le p \le \infty$, there exists an unique $T^{-1}(\Sigma)$-measurable function $E(f)$ such that

$$\int_A g f \, d\mu = \int_A g E(f) \, d\mu$$

for every $T^{-1}(\Sigma)$-measurable function g, for which the left integral exists. The function $E(f)$ is called the conditional expectation of f with respect to the subalgebra $T^{-1}(\Sigma)$. As an operator of $L^p(\mu)$, E is the projection onto the closure of range of T and E is the identity on $L^p(\mu)$, $p \ge 1$ if and only if $T^{-1}(\Sigma) = \Sigma$. Detailed discussion of E is found in [1-4].

a) *Normal operator*

Let H be a Complex Hilbert Space. An operator T on H is called normal operator if $T^*T = TT^*$

b) *Quasi-normal operator*

Let H be a Complex Hilbert Space. An operator T on H is called Quasi-normal operator if $T T^* T = T^* T T$, ie, $T^* T$ commute with T

c) *Quasi p-normal operator [13]*

Let H be a Complex Hilbert Space. An operator T on H is called Quasi-normal operator if $T^*T\left(T + T^*\right) = \left(T + T^*\right)T^*T$

d) *2-Power -normal operator*

Let H be a Complex Hilbert Space. An operator T on H is called 2 power-normal operator if $T^2 T^* = T^* T^2$

e) *Class Q-operator [14]*

Let H be a Complex Hilbert Space. An operator T on H is called Quasi-normal operator if $T^{*2} T^2 = \left(T^* T\right)^2$

II. Related Work in the Field

The study of weighted composition operators on L^2 spaces was initiated by R. K. Singh and D. C. Kumar [5]. During the last thirty years, several authors have studied the properties of various classes of weighted composition operator. Boundedness of the composition operators in $L^p(\sum), (1 \le p < \infty)$ spaces, where the measure spaces are σ-finite, appeared already in [6]. Also boundedness of weighted operators on $C(X, E)$ has been studied in [7]. Recently S. Senthil, P. Thangaraju, Nithya M, Surya devi B and D. C. Kumar, have proved several theorems on n-normal , n-quasi-normal, k-paranormal, and (n,k) paranormal of composite multiplication operators on L^2 spaces [8-12]. In this

Ref

5. Singh, RK & Kumar, DC, Weighted composition operators, Ph.D. thesis, Univ. of Jammu (1985).

paper we investigate composite multiplication operators on $L^2(\mu)$-space become Quasi-P-Normal operators and n-Power class Q operator have been obtained in terms of radon-nikodym derivative f_0 .

III. Characterization on Composite Multiplication of Quasi P Normal Operators on L^2-Space

Notes

a) Proposition

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$.Then for $u \geq 0$

(i) $M^*_{u,T} M_{u,T} f = u^2 f_0 f$

(ii) $M_{u,T} M^*_{u,T} f = u^2 \circ T \cdot f_0 \circ T \cdot E(f)$

(iii) $M^n_{u,T}(f) = (C_T M_u)^n (f) = u_n \ (f \circ T^n)$, $u_n = u \circ T . u \circ T^2 . u \circ T^3 ....u \circ T^n$

(iv) $M^*_{u,T} f = u f_0 \cdot E(f) \circ T^{-1}$

(v) $M^{*^n}_{u,T} f = u f_0 \cdot E(u f_0) \circ T^{-(n-1)} \cdot E(f) \circ T^{-n}$

where $E(u f_0) \circ T^{-(n-1)} = E(u f_0) \circ T^{-1} \cdot E(u f_0) \circ T^{-2} .... E(u f_0) \circ T^{-(n-1)}$

Theorem 3.1

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then the following statements are equivalent

(i) $M_{u,T}$ is Quasi p-normal operator

$$u \circ T \ u^2 \circ T \ h \circ T \ f \circ T + h u E\left(h u^2 f\right) \circ T^{-1} = h u^2 \ u \circ T \ f \circ T + h^2 u^3 E(f)$$

Proof:

For $f \in L^2(\mu)$, $M_{u,T}$ is Quasi P-normal operator if

$\left(M_{u,T} + M^*_{u,T}\right)\left(M^*_{u,T} M_{u,T}\right) f = \left(M^*_{u,T} M_{u,T}\right)\left(M_{u,T} + M^*_{u,T}\right) f$ and we have,

$$\left(M_{u,T} + M^*_{u,T}\right)\left(M^*_{u,T} M_{u,T}\right) f = M_{u,T}\left(M^*_{u,T} M_{u,T}\right) f + M^*_{u,T}\left(M^*_{u,T} M_{u,T}\right) f$$

$$= M_{u,T} M^*_{u,T} \left(u \circ T \ f \circ T\right) + M^*_{u,T} M^*_{u,T} \left(u \circ T \ f \circ T\right)$$

$$= M_{u,T}\left[h u E\left(uf \circ T\right) \circ T^{-1}\right] + M^*_{u,T}\left[h u E\left(uf \circ T\right) \circ T^{-1}\right]$$

$$= M_{u,T}\left[h u^2 f\right] + M^*_{u,T}\left[h u^2 f\right]$$

$$= u \circ T\left(h u^2 f\right) \circ T + h u E\left(h u^2 f\right) \circ T^{-1}$$

$$= u \circ T \ u^2 \circ T \ h \circ T \ f \circ T + h u E\left(h u^2 f\right) \circ T^{-1}$$

Consider

$$\left(M^*_{u,T}\,M_{u,T}\right)\left(M_{u,T}+M^*_{u,T}\right)f=\left(M^*_{u,T}\,M_{u,T}\right)M_{u,T}\,f+\left(M^*_{u,T}\,M_{u,T}\right)M^*_{u,T}\,f$$

$$=\left(M^*_{u,T}\,M_{u,T}\right)\left(u\circ T\ f\circ T\right)+\left(M^*_{u,T}\,M_{u,T}\right)\left(h\,u\,E(f)\circ T^{-1}\right)$$

$$=M^*_{u,T}\ u\circ T\ \left(u\circ T\ f\circ T\right)\circ T+M^*_{u,T}\ u\circ T\ \left(h\,u\,E(f)\circ T^{-1}\right)\circ T$$

$$=h\,u\,E\left[u\circ T\ u\circ T^2\ f\circ T^2\right]\circ T^{-1}+h\,u\,E\left[u\circ T\ h\circ T\ u\circ T\ E(f)\right]\circ T^{-1}$$

$$=h\,u^2\ u\circ T\ f\circ T+h^2\ u^3\ E(f)$$

Suppose, $M_{u,T}$ is Quasi P-normal operator. Then

$$\left(M_{u,T}+M^*_{u,T}\right)\left(M^*_{u,T}M_{u,T}\right)f=\left(M^*_{u,T}M_{u,T}\right)\left(M_{u,T}+M^*_{u,T}\right)f$$

$$\Leftrightarrow u\circ T\ u^2\circ T\ h\circ T\ f\circ T+h\,u\,E\ \ h\,u^2\left(f\ \circ T\right)^{-1}=h\,u^2\ u\circ T\ f\circ T+h^2\ u^3\ E(f)$$

Theorem 3.2

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then the following statements are equivalent

(i) $M^*_{u,T}$ is Quasi p-normal operator

$$h^2\,u^3\,E(f)\circ T^{-1}+h\circ T^2\ u\circ T\ u^2\circ T^2\,E(f)\circ T$$

(ii) $=h\circ T\ u^2\circ T\ E(h)\ E(u)E(f)\circ T^{-1}+h\circ T\ u\circ T\ u^2\circ T\ f\circ T$

Proof:

For $f\in L^2(\mu)$, $M^*_{u,T}$ is Quasi P-normal operator if

$$\left(M^*_{u,T}+M_{u,T}\right)\left(M_{u,T}\,M^*_{u,T}\right)f=\left(M_{u,T}\,M^*_{u,T}\right)\left(M^*_{u,T}+M_{u,T}\right)f$$

and then we have

$$\left(M^*_{u,T}+M_{u,T}\right)\left(M_{u,T}\,M^*_{u,T}\right)f=M^*_{u,T}\left(M_{u,T}\,M^*_{u,T}\right)f+M_{u,T}\left(M_{u,T}\,M^*_{u,T}\right)f$$

$$=M^*_{u,T}M_{u,T}\left[h\,u\,E(f)\circ T^{-1}\right]+M_{u,T}\,M_{u,T}\left[h\,u\,E(f)\circ T^{-1}\right]$$

$$=M^*_{u,T}\ u\circ T\left[h\,u\,E(f)\circ T^{-1}\right]\circ T+M_{u,T}\ u\circ T\left[h\,u\,E(f)\circ T^{-1}\right]\circ T$$

$$=M^*_{u,T}\left[u\circ T\ h\circ T\,u\circ T\ E(f)\right]+M_{u,T}\left[u\circ T\ h\circ T\,u\circ T\ E(f)\right]$$

Notes

Global Journal of Science Frontier Research (F) Volume XXIII Issue VI Version I Year 2023
76

$$= h\,u\,E\big(u\circ T\ h\circ T\,u\circ T\ E(f)\big)\circ T^{-1} + u\circ T\big[u\circ T\ h\circ T\,u\circ T\ E(f)\,\big]\circ T$$

$$= h^{2}\,u^{3}\,E(f)\circ T^{-1} + h\circ T^{2}\,u\circ T\ u^{2}\circ T^{2}\,E(f)\circ T$$

Notes

Consider

$$\left(M_{u,T}\,M^{*}{}_{u,T}\right)\left(M^{*}{}_{u,T} + M_{u,T}\right)f = \left(M_{u,T}\,M^{*}{}_{u,T}\right)M^{*}{}_{u,T}\ f + \left(M_{u,T}\,M^{*}{}_{u,T}\right)M_{u,T}\ f$$

$$= \left(M_{u,T}\,M^{*}{}_{u,T}\right)h\,u\,E(f)\circ T^{-1} + \left(M_{u,T}\,M^{*}{}_{u,T}\right)(u\circ T\ f\circ T)$$

$$= M_{u,T}\,h\,u\,E\left(h\,u\,E(f)\circ T^{-1}\right)\circ T^{-1} + M_{u,T}\,h\,u\,E\left(u\circ T\ f\circ T\right)\circ T^{-1}$$

$$= M_{u,T}\,h\,u\,E(h)\circ T^{-1}E(u)\circ T^{-1}E(f)\circ T^{-2} + M_{u,T}\,h\,u^{2}\,f$$

$$= u\circ T\left(h\,u\,E(h)\circ T^{-1}\,E(u)\circ T^{-1}\,E(f)\circ T^{-2}\right)\circ T + u\circ T\,(h\,u^{2}f)\circ T$$

$$= h\circ T\ u^{2}\circ T\ E(h)\ E(u)\,E(f)\circ T^{-1} + h\circ T\ u\circ T\ u^{2}\circ T\ f\circ T$$

Suppose $M^{*}{}_{u,T}$ is Quasi p-normal operator. Then

$$\left(M^{*}{}_{u,T} + M_{u,T}\right)\left(M_{u,T}\,M^{*}{}_{u,T}\right)f = \left(M_{u,T}\,M^{*}{}_{u,T}\right)\left(M^{*}{}_{u,T} + M_{u,T}\right)f$$

$$\Leftrightarrow h^{2}\,u^{3}\,E(f)\circ T^{-1} + h\circ T^{2}\,u\circ T\ u^{2}\circ T^{2}\,E(f)\circ T$$

$$= h\circ T\ u^{2}\circ T\ E(h)\ E(u)\,E(f)\circ T^{-1} + h\circ T\ u\circ T\ u^{2}\circ T\ f\circ T$$

IV. Characterizations on N Power Class Q Composite Multiplication Operatos on L^2-Space

Theorem 4.1

Let the $M_{u,T}$ be a composite multiplication operator on $L^{2}(\mu)$. Then $M_{u,T}$ is n power class Q composite multiplication operator if and only if

$$h\,u\,E(h)\circ T^{-1}\,E(u)\circ T^{-1}\,E(u_{2n})\circ T^{-2}\,f\circ T^{2n-2}$$
$$= h\,u\,E(u_{n})\circ T^{-1}\,h\circ T^{n-1}\,u\circ T^{n-1}\,E(u_{n})\circ T^{n-2}\,f\circ T^{2n-2}$$

Proof:
Now Consider,

$$M^{*2}{}_{u,T}\,M^{2n}{}_{u,T}\ f = M^{*2}{}_{u,T}\left[u_{2n}f\circ T^{2n}\right]$$

where $u_{2n} = u\circ T^{2}\ u\circ T^{4}\,\ldots\ldots\ldots\ldots\ldots\ u\circ T^{2n}$

$$= M^*_{u,T}\left(h\,u\,E\left(u_{2n}\,f\circ T^{2n}\right)\circ T^{-1}\right)$$

$$= M^*_{u,T}\,h\,u\,E(u_{2n})\circ T^{-1}\,f\circ T^{2n-1}$$

$$= h\,u\,E\left(h\,u\,E(u_{2n})\circ T^{-1}\,f\circ T^{2n-1}\right)\circ T^{-1}$$

$$= h\,u\,E(h)\circ T^{-1}\,E(u)\circ T^{-1}\,E(u_{2n})\circ T^{-2}\,f\circ T^{2n-2}$$

Notes

Next we consider,

$$\left(M^*_{u,T}M^n_{u,T}\right)^2 f = \left(M^*_{u,T}M^n_{u,T}\right)\left(M^*_{u,T}M^n_{u,T}\right)f$$

$$= \left(M^*_{u,T}M^n_{u,T}\right)M^*_{u,T}\left(u_n\,f\circ T^n\right)$$

where $u_n = u\circ T\; u\circ T^2 \ldots\ldots\ldots\ldots\ldots\; u\circ T^n$

$$= \left(M^*_{u,T}M^n_{u,T}\right)h\,u\,E\left(u_n\,f\circ T^n\right)\circ T^{-1}$$

$$= \left(M^*_{u,T}M^n_{u,T}\right)h\,u\,E(u_n)\circ T^{-1}\,f\circ T^{n-1}$$

$$= M^*_{u,T}\;u_n\left(h\,u\,E(u_n)\circ T^{-1}\,f\circ T^{n-1}\right)\circ T^n$$

$$= M^*_{u,T}\;u_n\,h\circ T^n\,u\circ T^n\,E(u_n)\circ T^{n-1}\,f\circ T^{2n-1}$$

$$= h\,u\,E\left(u_n\,h\circ T^n\,u\circ T^n\,E(u_n)\circ T^{n-1}\,f\circ T^{2n-1}\right)\circ T^{-1}$$

$$= h\,u\,E(u_n)\circ T^{-1}\,h\circ T^{n-1}\,u\circ T^{n-1}\,E(u_n)\circ T^{n-2}\,f\circ T^{2n-2}$$

Given $M_{u,T}$ is n power class Q composite multiplication operator

$$\Leftrightarrow M^{*2}_{u,T}\,M^{2n}_{u,T}\;f = \left(M^*_{u,T}M^n_{u,T}\right)^2 f$$

$$\Leftrightarrow h\,u\,E(h)\circ T^{-1}\,E(u)\circ T^{-1}\,E(u_{2n})\circ T^{-2}\,f\circ T^{2n-2}$$

$$= h\,u\,E(u_n)\circ T^{-1}\,h\circ T^{n-1}\,u\circ T^{n-1}\,E(u_n)\circ T^{n-2}\,f\circ T^{2n-2}$$

Theorem 4.2

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then $M^*_{u,T}$ is n power class Q composite multiplication operator if and only if

$$u\circ T\;u^2\circ T^2\,h\circ T^2\,E(u\,h)\circ T^{-(2n-3)}\,E(f)\circ T^{-(2n-2)}$$

$$= u^2\circ T\;h\circ T\;E(u\,h)\circ T^{-(n-2)}\,h\circ T^{-(n-2)}\,E(u\,h)\circ T^{-(2n-3)}\,E(f)\circ T^{-(2n-2)}$$

Notes

Proof:
Now if we consider

$$M^2{}_{u,T} M^{*2n}{}_{u,T}\ f = M^2{}_{u,T}\left(h\, u\, E(h\, u)\circ T^{-(2n-1)}\ E(f)\circ T^{-2n}\right)$$

$$= M_{u,T}\left(u\circ T\left(h\, u\, E(h\, u)\circ T^{-(2n-1)}\ E(f)\circ T^{-2n}\right)\circ T\right)$$

$$= M_{u,T}\left(h\circ T\ \ u^2\circ T\ E(h\, u)\circ T^{-(2n-2)}\, E(f)\circ T^{-(2n-1)}\right)$$

$$= u\circ T\ \left(h\circ T\ \ u^2\circ T\ E(h\, u)\circ T^{-(2n-2)}\, E(f)\circ T^{-(2n-1)}\right)\circ T$$

$$= u\circ T\ u^2\circ T^2\ h\circ T^2\ E(u\, h)\circ T^{-(2n-3)}\, E(f)\circ T^{-(2n-2)}$$

and we consider

$$\left(M_{u,T}\, M^{*n}{}_{u,T}\right)^2 f = \left(M_{u,T}\, M^{*n}{}_{u,T}\right)\left(M_{u,T}\, M^{*n}{}_{u,T}\right) f$$

$$= \left(M_{u,T}\, M^{*n}{}_{u,T}\right) M_{u,T}\ \ u\, h\, E(u\, h)\circ T^{-(n-1)}\ E(f)\circ T^{-n}$$

$$= \left(M_{u,T}\, M^{*n}{}_{u,T}\right) u\circ T\left(u\, h\, E(u\, h)\circ T^{-(n-1)}\ E(f)\circ T^{-n}\right)\circ T$$

$$= M_{u,T}\ M^{*n}{}_{u,T}\ \left(u^2\circ T\ \ h\circ T\ E(u\, h)\circ T^{-(n-2)}\ \ E(f)\circ T^{-(n-1)}\right)$$

$$= M_{u,T}\ u\, h\, E(u\, h)\circ T^{-(n-1)}\ E\left(u^2\circ T\ \ h\circ T\ E(u\, h)\circ T^{-(n-2)}\ \ E(f)\circ T^{-(n-1)}\right)\circ T^{-n}$$

$$= M_{u,T}\ \left(u\, h\, E(u\, h)\circ T^{-(n-1)}\ u^2\circ T^{-(n-1)}\ h\circ T^{-(n-1)}\, E(u\, h)\circ T^{-(2n-2)}\ E(f)\circ T^{-(2n-1)}\right)$$

$$= u\circ T\left(\ u\, h\, E(u\, h)\circ T^{-(n-1)}\ u^2\circ T^{-(n-1)}\ h\circ T^{-(n-1)}\, E(u\, h)\circ T^{-(2n-2)}\ E(f)\circ T^{-(2n-1)}\right)\circ T$$

$$= u^2\circ T\ \ h\circ T\ E(u\, h)\circ T^{-(n-2)}\ h\circ T^{-(n-2)}\ E(u\, h)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$

Since $M_{u,T}$ is a Composite multiplication operator, by definition

$$\Leftrightarrow\ M^2{}_{u,T}\, M^{*2n}{}_{u,T}\ f = \left(M_{u,T}\, M^{*n}{}_{u,T}\right)^2 f$$

$$\Leftrightarrow\ u\circ T\ u^2\circ T^2\ h\circ T^2\ E(u\, h)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$
$$= u^2\circ T\ \ h\circ T\ E(u\, h)\circ T^{-(n-2)}\ h\circ T^{-(n-2)}\ E(u\, h)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$

References Références Referencias

1. Campbell, J & Jamison, J, On some classes of weighted composition operators, Glasgow Math.J.vol.32, pp.82-94, (1990).

2. Embry Wardrop, M & Lambert, A, Measurable transformations and centred composition operators, Proc.Royal Irish Acad, vol.2(1), pp.23-25 (2009).
3. Herron, J, Weighted conditional expectation operators on L^p -spaces, UNC charlotte doctoral dissertation.
4. Thomas Hoover, Alan Lambert and Joseph Quinn, The Markov process determined by a weighted composition operator, Studia Mathematica, vol. XXII (1982).
5. Singh, RK & Kumar, DC, Weighted composition operators, Ph.D. thesis, Univ. of Jammu (1985).
6. Singh, RK Composition operators induced by rational functions, Proc. Amer. Math. Soc., vol.59, pp.329-333(1976).
7. Takagi, H & Yokouchi, K, Multiplication and Composition operators between two L^p-spaces, Contem. Math., vol.232, pp.321-338 (1999).
8. Senthil S, Thangaraju P & Kumar DC, "Composite multiplication operators on L^2-spaces of vector valued Functions", Int. Research Journal of Mathematical Sciences, ISSN 2278-8697, Vol.(4), pp.1 (2015).
9. Senthil S, Thangaraju P & Kumar DC, "k-*Paranormal, k-Quasi-*paranormal and (n, k)- quasi-*paranormal composite multiplication operator on L^2 –spaces, British Journal of Mathematics & Computer Science, 11(6): 1-15, 2015, Article no.BJMCS.20166, ISSN: 2231-0851 (2015).
10. Senthil S, Thangaraju, P & Kumar, DC, n-normal and n-quasi-normal composite multiplication operator on L^2 -spaces, Journal of Scientific Research & Reports,8(4),1-9 (2015).
11. Senthil S, Nithya M and Kumar DC, "(Alpha, Beta)-Normal and Skew n-Normal Composite Multiplication Operator on Hilbert Spaces" International Journal of Discrete Mathematics, ISSN: 2578-9244 (Print); ISSN: 2578-9252; Vol.4 (1), pp. 45-51 (2019).
12. Senthil S, Nithya M and Kumar DC, " Composite Multiplication Pre-Frame Operators on the Space of Vector-Valued Weakly Measurable Functions" Global Journal of Science Frontier Research: F Mathematics and Decision Sciences Vol.20 (7), pp.1-12, ISSN: 2249-4626 & Print ISSN: 0975-5896 (2020)
13. Bhattacharya D and Prasad N, "Quasi-P Normal operators - linear operators on Hilbert space for which T+T* and T*T commute", Ultra Scientist, vol.24(2A), pp. 269-272 (2012).
14. Adnan A and Jibril AS, "On operators for which $T^{*2}T^2 = (T^*T)^2$, International Mathematical forum, vol. 5(46), pp.2255 – 2262 (2010).
15. Panayappan S and Sivamani N, "On n power class (Q) operators", Int. Journal of Math. Analysis, vol.6(31), pp.1513-1518 (2012).

Notes