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Distinguished Couple of Integer Right Triangles and Canada Numbers

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DISTINGUISHED COUPLE OF INTEGER RIGHT TRIANGLES AND CANADA NUMBERS

Ref

1. Sierpinski W, Pythagorean triangles, Dover publications, INC, New York, 2003.

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I. Introduction

The theory of numbers is a fascinating area of mathematics. Right integer triangles have attracted the attention of many mathematicians and math enthusiasts because it is a treasure house where finding many hidden connections is like going on a treasure hunt. Refer to [1]-[3] for various fascinating challenges. In addition to polygonal numbers, we also have the Jarasandha numbers, Nasty numbers, Dhuruva numbers, and Canada Numbers, which are all intriguing patterns of numbers. These figures are displayed in [4]-[9]. Special Pythagorean triangles linked to Nasty and polygonal numbers are derived in [10]–[15].

In this writing, we look for a distinguished couple of right integer triangles where the difference in their perimeters in each pair is four times the Canada numbers.

II. Basic Definitions

Definition: 1

The ternary quadratic Diophantine equation given by $s^2+t^2=r^2$ is known as Integer right equation, where s, t and r are natural numbers and denotes it by $\Delta(s,t,r)$. Also, in $\Delta(s,t,r): s^2+t^2=r^2$, s and t are called its legs and r its hypotenuse.

Definition: 2

The most cited solution of the Integer right equation is

$$s=a^2-b^2,\ t=2ab,\ r=a^2+b^2,$$

where $a>b>0$. If a and b have opposing parities and $\gcd(a,b)=1$, then this solution is referred to as primitive.

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Definition: 3

Canada numbers are those n such that the sum of the squares of the digits of n is equal to the sum of the non- trivial divisors of n, i.e., $\sigma(n)-n-1$.

The Canada numbers are 125, 581, 8549 and 16999.

The name of these numbers is due to the fact they were defined by some mathematicians from Manitoba University to celebrate the 125^{th} anniversary of Canada.

Notes

III. Materials and Methods

Let Δ_1 and Δ_2 be two distinct right integer triangles with generators $a, c\ (a>c>0)$, $b, c\ (b>c>0)$ respectively. Let P_1 and P_2 be the perimeters and Λ_1 *and* Λ_2 be the areas of Δ_1 and Δ_2 such that $P_1 - P_2 = 4$ times the 1^{st} Canada number 125. The equation derived from the relationship above is

$$(2a+c)^2-(2b+c)^2=1000 \tag{1}$$

It is observed after completing numerical calculations that there are 20 different values for $a, c,$ and b satisfied (1) provided $a+b+c=$ Canada number

The values of a, c, b, P_1 and P_2 are shown in Table I below for clarity and simplicity.

Table I

S. No.	a	c	b	P_1	P_2	$\frac{P_1 - P_2}{4}$
1.	44	39	42	7304	6804	125
2.	45	37	43	7380	6880	125
3.	46	35	44	7452	6952	125
4.	47	33	45	7520	7020	125
5.	48	31	46	7584	7084	125
6.	49	29	47	7644	7144	125
7.	50	27	48	7700	7200	125
8.	51	25	49	7752	7252	125
9.	52	23	50	7800	7300	125
10.	53	21	51	7844	7344	125
11.	54	19	52	7884	7384	125
12.	55	17	53	7920	7420	125
13.	56	15	54	7952	7452	125
14.	57	13	55	7980	7480	125
15.	58	11	56	8004	7504	125
16.	59	9	57	8024	7524	125
17.	60	7	58	8040	7540	125
18.	61	5	59	8052	7552	125
19.	62	3	60	8060	7560	125
20.	63	1	61	8064	7564	125

Thus, it can be observed that there are 20 couples of right integer triangles, where each couple's difference in perimeters equals four times the first Canada number (125). Of these 20 couples, ten are non-primitive, six are primitive, and four are couples, where one is a primitive triangle and the other is non-primitive.

The following Table II illustrates a similar observation of other Canada numbers:

Notes

Table II

Canada Number	Couples of Right integer Triangles	Couples of primitive Right integer Triangles	Couples of non-primitive Right integer Triangles	Couples of primitive and non-primitive Right integer Triangles
581	96	28	50	18
8549	1424	337	733	354
16999	2833	903	1438	492

IV. Conclusion

In this article, we propose a couple of integer right triangles whose perimeter differences are each equal to four times the Canada number. We also provide the number of couples containing primitive and non-primitive integer right triangles. In conclusion, one can look for relationships between distinguished couples of integer right triangles and other unique numbers and number patterns.

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