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Exploring Repetitive Integer Patterns in the Complex Roots of Homogeneous Polynomials

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EXPLORINGREPETITIVEINTEGERPATTERNSINTHECOMPLEXROOTSOFHOMOGENEOUSPOLYNOMIALS

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Exploring Repetitive Integer Patterns in the Complex Roots of Homogeneous Polynomials

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Abstract- In similar 4th degree polynomials, certain roots exhibit a pattern where an integer serves as both a negative factor of the polynomial's constant and the value of the imaginary component of the root. This integer, called the 'negative base multiple,' appears consistently across multiple sets, which we term 'iterative imaginary number sets.' By increasing initial γ values starting at n=3, this pattern is observed for entire sets of multiples.

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I. Introduction

a) Opening

Mathematics is at its root, a study of patterns using constraints and established axioms to establish new techniques and ultimately, gain a deeper understanding of the numbers which live all around us. Among these numbers, complex numbers, often referred to as imaginary numbers, offer solutions where real numbers fall short. Applications of these number types appear in many real-world applications including quantum computing, medical imaging, financial mathematics and optical engineering, amongst many other various fields. This article will focus on identifying and analyzing patterns in the integers that appear repetitively in both the inputs of the polynomials and the imaginary components of their roots or zeros.

b) Scope of Paper

In the scope of this work, we aim to establish the idea of iterative imaginary number sets, or numbers that are both real and imaginary, matching x-values of roots for certain 4th degree polynomials. The similar polynomial expressions contain four interchangeable variables: two that makeup the complete iterative set, which we call multiple sets for each similar polynomial, one which is a converging negative constant and one which is a converging positive coefficient. The polynomial expressions themselves are similar:

$$x^4 + \alpha x^3 + \beta x^2 + \gamma x - c = 0$$

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Where, α is the first variable of the multiple set, γ is the second variable of the multiple set, c is some negative constant of which the x-value is a negative factor of, and β is some positive coefficient value, beginning at $\beta = 2$.

II. Methodology

a) Background

In fourth-degree polynomials expressions, specific patterns arise where the roots include an imaginary value that corresponds to a negative factor of the polynomial's constant term. This pattern repeats across multiples of these negative constants. By increasing initial γ values beginning at n=3, we find the pattern exists for entire sets of multiples of the negative constant, which is the same value as the polynomials negative square root zero. and what we call 'iterative imaginary number sets', a formal name given to the repetitive integers that serves as both a negative factor of the polynomial's constant and the imaginary component of the root. You will see over the next few sections of equations, that iterative imaginary numbers like -3, -7.-11, -15 and -19 share a duality when configured into this general polynomial expression, as being *both* the x-value within the negative square root (imaginary value) of the polynomials root *and* a negative factor of the polynomials negative constant, while also representing the base of general multiplicative set being tested by the polynomial.

b) Multiples Sets in the Pattern

Set A is the example for -7:

Set A

Multiples	α	γ
M1	1	7
M2	2	14
M3	3	21
M4	4	28
M5	5	35
M6	6	42
M7	7	49
M8	8	56
M9	9	63

Here, for all $\alpha \wedge \gamma \in A$, when expressed as roots of the constricted polynomial:

$$x^4 + \alpha x^3 + 2x^2 + \gamma x - 35 = 0$$

Notes

Result in the same constant complex x-values of $x = \pm i\sqrt{7}$ for every α, γ *in* A & $c = -35$. We cannot alter the constant for the set of -7 (Set *A*) and still effectively see the pattern result from same statement, meaning that for the multiples of -7, the negative constant can only be -35 & the base multiple will always be $x = \pm i\sqrt{7}$.

Notes

The pattern arises from the general polynomial form then in that the negative base multiple (M1 γ for each set) is also the same integer inside the negative square root, when finding zeroes of the polynomial. The imaginary unit and the real number share this commonality for every multiple set, for the first 9 positive multiple pairs of each multiple set. The α of each multiple set remains the same, acting as both an independent variable and the multiplier by the base multiple for each pair to create each γ value.

The imaginary zeroes of the polynomial will always be the same value as the negative base multiple, defined here as Iterative Imaginary Zeroes.

c) Algebra for Set A of Iterative Multiples

Proof for Set *A*, Negative Base Multiple of -7 (finding *x* for the complex result only)

(M1)

$$x^4 + x^3 + 2x^2 + 7x - 35 = 0$$

$$((x^2 - 7)(x^2 + x - 5) = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

(M2)

$$x^4 + 2x^3 + 2x^2 + 14x - 35 = 0$$

$$((x^2 - 7)(x^2 + 2x - 5) = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

(M3)

$$x^4 + 3x^3 + 2x^2 + 21x - 35 = 0$$

$$((x^2 - 7)(x^2 + 3x - 5) = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

(M4)

$$x^4 + 4x^3 + 2x^2 + 28x - 35 = 0$$

$$((x^2 - 7)(x-)(x + 5) = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

(M5)

$$x^4 + 5x^3 + 2x^2 + 35x - 35 = 0$$

$$((x^2 - 7)(x^2 + 5x - 5) = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

(M6)

$$x^4 + 6x^3 + 2x^2 + 42x - 35 = 0$$

$$((x^2 - 7)(x^2 + 6x - 5) = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

(M7)

$$x^4 + 7x^3 + 2x^2 + 49x - 35 = 0$$

$$((x^2 - 7)(x^2 + x - 5) = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

(M8)

$$x^4 + 8x^3 + 2x^2 + 56x - 35 = 0$$

$$((x^2 - 7)(x^2 + 7x - 5) = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

(M9)

$$x^4 + 9x^3 + 2x^2 + 63x - 35 = 0$$

$$((x^2 - 7)(x^2 + 9x - 5) = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

So, for all $\alpha \,\&\, \gamma \in A$ when expressed as zeros of the polynomial:

$$x^4 + \alpha x^3 + 2x^2 + \gamma x - 35 = 0$$

Notes

There exists a pattern in both the complex negative square root value and one of the factors of the negative constant both being -7. This same pattern exists for at least four other negative integers, five in total: -3, -7, -11, -15, & -19. In similarly structured, homogenous polynomials, these negative integers alongside their multiple sets result in the same pattern as shown above.

Notes

d) Different Negative Base Multiples of Homogeneous Polynomial Expressions

In a slightly altered form of the same polynomial, the pattern also exists for -3 in the same manner. Using the altered 4th degree polynomial:

$$x^4 + \alpha x^3 + x^2 + \gamma x - 6 = 0$$

Set B

Multiples	α	γ
M1	1	3
M2	2	6
M3	3	9
M4	4	12
M5	5	15
M6	6	18
M7	7	21
M8	8	24
M9	9	27

Results in $= \pm i\sqrt{3}$ and -3 is the negative factor of -6, just as the polynomial

$$x^4 + \alpha x^3 + 2x^2 + \gamma x - 35 = 0$$

Results in $\pm i\sqrt{7}$ and -7 is the negative factor of -35.

Similar multiple sets with homogenous expressions of the general polynomial also exist for -11, -15 & -19.

$$x^4 + \alpha x^3 + 3x^2 + \gamma x - 88 = 0$$

Set C

Multiples	α	γ
M1	1	11
M2	2	22
M3	3	33
M4	4	44
M5	5	55
M6	6	66
M7	7	77
M8	8	88
M9	9	99

Results in $= \pm i\sqrt{11}$ and -11 is the negative factor of -88.

$$x^4 + \alpha x^3 + 3x^2 + \gamma x - 180 = 0$$

Set D

Multiples	α	γ
M1	1	15
M2	2	30
M3	3	45
M4	4	60
M5	5	75
M6	6	90
M7	7	105
M8	8	120
M9	9	135

Results in $= \pm i\sqrt{15}$ and -15 is the negative factor of -180.

$$x^4 + \alpha x^3 + 5x^2 + \gamma x - 266 = 0$$

Notes

Notes

Set D

Multiples	$\boldsymbol{\alpha}$	$\boldsymbol{\gamma}$
M1	1	19
M2	2	38
M3	3	57
M4	4	76
M5	5	95
M6	6	114
M7	7	133
M8	8	152
M9	9	171

Results in $= \pm i\sqrt{19}$ and -19 is the negative factor of -266.

III. Conclusionary Discussion

This study identified a unique pattern where specific integers manifest as both factors in the polynomial's constants and as values within the imaginary components of the roots. While the procedure can be extended to higher-degree polynomials and additional iterative imaginary zeros, this initial investigation establishes the foundation for further exploration of the patterns. The patterns identified here, while simple in scope, contribute to the broader understanding of polynomial structures and their roots.

While this paper has focused on identifying patterns in fourth-degree polynomials, future research could explore whether similar patters exist for higher-degree polynomials and more complex root structures. Additionally, further work could investigate potential applications of these patterns in other areas of mathematics or applied fields.